$x=\frac{a^2-b^2}{y}=\frac{500}{y}=35.559825.\quad x+y=49.620636,\ x-y=21.499014.$

$AB=\sqrt{(AC^2-BC^2)}=\sqrt{[a^2-(x+y)^2]}=33.731178.$

Solutions were also received from *G. B. M. ZERR, J. F. W. SCHEFFER, L. B. FRAKER*, and *J. K. ELLWOOD*.

25. Proposed by L. B. FRAKER, Weston, Ohio.

The sides of a quadrilateral board are $AB=7$ inches, $BC=15$ inches, $CD=21$ inches, and $DA=13$ inches; radius of incribed circle is 6 inches. (1) What are dimensions of the largest rectangular board that can be cut out of the given board, (2) largest square, (3) largest equilateral triangle? (Please solve without use of the calculus.)

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $Aa=Ad=x$, $Ba=Bb=y$, $Cb=Cc=z$, $Dc=Dd=w$,

then $x+y=AB=7=6\cot\frac{1}{2}A+6\cot\frac{1}{2}B\ldots.(1)$,

$y+z=BC=15=6\cot\frac{1}{2}B+6\cot\frac{1}{2}C\ldots.(2)$,

$z+w=CD=21=6\cot\frac{1}{2}C+6\cot\frac{1}{2}D\ldots.(3)$,

$w+x=DA=13=6\cot\frac{1}{2}D+6\cot\frac{1}{2}A\ldots.(4)$.

$\therefore\quad 28=6\cot\frac{1}{2}A+6\cot\frac{1}{2}B+6\cot\frac{1}{2}C+6\cot\frac{1}{2}D\ldots.(5)$.

$(5)-\{(1)+(4)\}$ gives $4=3\cot\frac{1}{2}C-3\cot\frac{1}{2}A$

$\therefore\quad 4\sin\frac{1}{2}A\sin\frac{1}{2}C=3\sin\frac{1}{2}A\cos\frac{1}{2}C-3\cos\frac{1}{2}A\sin\frac{1}{2}C\ldots.(6)$.

$(5)-\{(3)+(4)\}$ gives $1=\cos\frac{1}{2}D-\cos\frac{1}{2}B$

$\therefore\quad \sin\frac{1}{2}D\sin\frac{1}{2}B=\sin\frac{1}{2}B\cos\frac{1}{2}D-\cos\frac{1}{2}B\sin\frac{1}{2}D\ldots.(7)$.

From DB^2 we get $45\cos C-13\cos A=32$.

$\therefore\quad 13\sin^2\frac{1}{2}A=45\sin^2\frac{1}{2}C\ldots.(8)$.

$45\cos^2\frac{1}{2}C-13\cos^2\frac{1}{2}A=32\ldots.(9)$.

From AC^2 we get $13\cos D-5\cos B=8$.

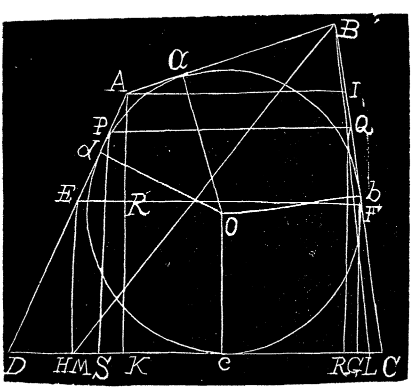
$\therefore\quad 13\sin^2\frac{1}{2}D=5\sin^2\frac{1}{2}B\ldots.(10)$.

$13\cos^2\frac{1}{2}D-5\cos^2\frac{1}{2}B=8\ldots.(11)$.

From (6), (7), (8), (9), (10), (11) we get

$A=134°\ 45'\ 35''$, $B=106°\ 15'\ 39''$, $C=D=59°\ 29'\ 23''$.

Now the greatest equilateral triangle is the one which has the greatest side. Since $D=C$ only two cases confront us, the one is to draw a line from B making an angle of 60° with DC, the other to draw a line from some point in DC making an angle of 6° with BC. The former line is found to be very little the longer. Let BL be this line, then we have $15 : BL=\sin 120° : \sin 59°\ 29'\ 23''$.



$\therefore\quad BL=$side of triangle$=14.92$ inches. $\therefore\quad BLM=$triangle. Also since $D=C$, the greatest square has its side coincident with DC. Hence $\frac{21-x}{2} : x=\cos 59°\ 29'\ 23'' : \sin 59°\ 29'\ 23''$.

$\therefore\quad x=$side of square$=9.64$ inches, and $PQRS=$square. Also since $D=C$, the side of the greatest rectangle will

coincide with DC. Draw AI parallel and AK perpendicular to DC and let $EFGH$ be the rectangle.

Then $\frac{1}{2}(AI+DC)\times AK=EF\times FG+FG\times GC+\frac{1}{2}(AI+EF)(AK-KR')$. But $AK=11.2$ inches, $AI=7.8$ inches. $\therefore$ $588=28\,EF+33FG$.

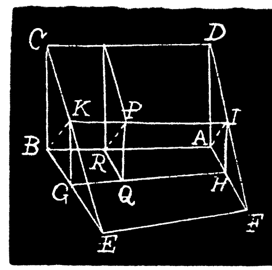
$\therefore$ for maximum $28\,EF=33\,FG$. $\therefore$ $EF=10.5$ inches, FG 8.91 inches.

26. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

$ABCD$ represents a triangle, and $ABEF$ a trapezoid which is perpendicular to the rectangle, both figures having the side AB common to each other, and ADF and BCE forming two right triangles perpendicular to the rectangle $ABCD$. To determine the conoidal surface $CDFE$ so as to satisfy the condition that any plane laid through AB will interest it in a straight line. Also find volume of the surface thus formed.

Solution by the PROPOSER.

Let $BC=AD=h$, $AB=a$, $BE=b$, $AF=c$. Let P represent a point in the surface, and put $AR=x$, $RQ=y$, $PQ=z$



The triangles BGK, PQR, and AHI are similar, and we may now put $AH=ny$, $IH=nz$, $BG=my$, $KG=mz$; but $h:mz=b-my$; $h:nz=c:c-nz$, whence $m=\dfrac{bh}{hy+bz}$, $n=\dfrac{ch}{hy+cz}$.

In the trapezoid $AHGB$, we now have $AB=a$, $AH=\dfrac{chy}{hy+cz}$, $BG=\dfrac{bhy}{hy+bz}$, $AR=x$, $RQ=y$. $\therefore$ $(AH+y)x+(BG+y)(a-x)=(AH+BG)a$.

Substituting, clearing of fractions, and arranging, we find for the equation of the surface

$$abcz^2+a(b+c)hyz-(b-c)h^2xy+ah^2y^2-abchz-ach^2y=0.$$

Let us now denote $\angle CBK=\angle DAI$ by θ, and angles BCK and ADI represent by C and D. For the volume we have

$$\tfrac{1}{6}ah^2\int_0^{\frac{\pi}{2}}\left[\frac{\sin^2 C}{\sin^2(C+\theta)}+\frac{\sin^2 D}{\sin^2(D+\theta)}+\frac{\sin C\sin D}{\sin(C+\theta)\sin(D+\theta)}\right]d\theta=\tfrac{1}{6}ah^2\left[\tan C+\tan D+\frac{\tan C\tan D}{\tan C-\tan D}\log\frac{\tan C}{\tan D}\right]$$

but $\tan C=\dfrac{b}{h}$, $\tan D=\dfrac{c}{h}$;

$$\therefore\ \text{volume}=\tfrac{1}{6}ah\left[b+c+\frac{bc}{b-c}\log\frac{b}{c}\right].$$

27. Proposed by ADOLPH BAILOFF, Durand Wisconsin.

A line BF, that bisects an angle exterior to the vertical angle of an isosceles triangle is parallel to the base AC.

Solution by Mrs. MARY E. HOGSETT, Danville, Kentucky: P. S. BERG, Apple Creek, Ohio, Professors JOHN FAUGHT, Bloomington, Indiana; and M. A. GRUBER, War Department, Washington, D. C.